# ПATIBIA UПIVERSITY <br> OF SCIEחCE AחD TECHחOLOGY <br> FACULTY OF HEALTH AND APPLIED SCIENCES 

DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics |  |
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| QUALIFICATION CODE: 07BSAM | LEVEL: 6 |
| COURSE CODE: MAP602S | COURSE NAME: MATHEMATICAL PROGRAMMING |
| SESSION: NOVEMBER 2022 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| FIRST OPPORTUNITY QUESTION PAPER |  |
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| EXAMINERS | MR. B.E OBABUEKI, MR J AMUNYELA |
| MODERATOR: | PROFESSOR ADETAYO EEGUNJOBI |

## INSTRUCTIONS

1. Answer ALL questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Excluding this front page)

## Question 1 (10 marks)

A cobbler makes three types of shoes: stiletto, casual and park. Each pair of stiletto takes 8 hours to fabricate, 5 hours to sand and 6 hours to couple. Each pair of casual takes 6 hours to fabricate, 4 hours to sand and 2 hours to couple. Each pair of park requires 5 hours of fabrication, 2 hours of sanding and 4 hours of coupling. The cobbler has 96 hours for fabrication, 44 hours for sanding and 58 hours for coupling. The profit margins are N\$38 per pair of stiletto, N\$26 per pair of casual and N\$22 per pair of park. Model this information into a linear programming problem. Declare your variables unambiguously and name the constraints. DO NO SOLVE.

## Question 2 (13 marks)

Solve the following linear programming model graphically:
Minimize $H=15 a+12 b$
Subject to $\quad 6 a+6 b \geq 36$

$$
3 a+9 b \geq 27
$$

$$
b \leq 3
$$

$$
a \leq 10
$$

$$
\begin{equation*}
a ; b \geq 0 \tag{13}
\end{equation*}
$$

Use 1 cm to I unit for each of the axes.

## Question 3 (28 marks)

Consider the following L-P model:
Minimize $\quad C=40 a+60 b+48 d$
Subject to $5 \mathrm{a}+3 b+4 d \geq 7$
$2 a+12 b+8 d \geq 21$
$a \geq 0 ; b \geq 0 ; d \geq 0$
3.1 Write down the dual of the model.
3.2 Solve the dual model.
3.3 Suppose the solution of the dual model is $x=4 ; y=4 ; t_{1}=12 ; t_{2}=0 ; t_{3}=0 ; C=112$.

Use this solution to determine the solution of the given primal model.
(9)

## Question 4 (18 marks)

Consider the following L-P model:
Minimize $Q=2 x+4 y+5 z+3 t$
Subject to $-x-2 y+2 z \geq 40$
$3 x \quad+2 z+t \leq 100$
$x-2 y-z+4 t \geq 50$
$x ; y ; z ; t \geq 0$
4.1 Identify all the non-basic variables in the model.
4.2 Express $H=A_{1}+A_{3}$ in terms of the non-basic variables.
4.3 Write down the initial tableau for the phase 1 of the two-phase method.
4.4 Given that the final tableau of phase 1 is

| $\mathbf{x}$ | $\mathbf{y}$ | $\mathbf{z}$ | $\mathbf{t}$ | $\mathbf{s 1}$ | $\mathbf{s 2}$ | $\mathbf{s 3}$ | A1 | A3 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | -2 | 2 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | 40 |
| 31 | 22 | 0 | 0 | 9 | 8 | 2 | -9 | -2 | 0 | 340 |
| 1 | -6 | 0 | 8 | -1 | 0 | -2 | 1 | 2 | 0 | 140 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 1 | 0 |

and that the original objective function is expressed in terms of non-basic variables for phase 2 as $8 Q=33 x+90 y+23 s_{1}+3 s_{3}+1220$, determine the solution of the given L-P model.

## Question 5 (20 marks)

Consider the following transportation table:

|  | Destination 1 | Destination 2 | Destination 3 | Destination 4 | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source 1 | 10 | 8 | 20 | 11 | $\mathbf{2 0}$ |
| Source 2 | 12 | 9 | 7 | 20 | $\mathbf{2 5}$ |
| Source 3 | 6 | 14 | 16 | 18 | $\mathbf{1 5}$ |
| Demand | $\mathbf{1 0}$ | $\mathbf{1 5}$ | $\mathbf{1 5}$ | 20 |  |

5.1 Determine the initial transportation cost using the Least-cost method.
5.2 The following table is an estimate of the minimum cost of the transportation problem:


Use this table to determine the minimum cost for the transportation problem.

## Question 6 (11 marks)

Given the following assignment table, assign workers $A, B$, and $C$ to the tasks 1,2 , and 3 in such a way that assignment cost is at its minimum.

|  | Task 1 | Task 2 | Task 3 |
| :--- | :--- | :--- | :--- |
| Worker A | 450 | 420 | 490 |
| Worker B | 360 | 450 | 400 |
| Worker C | 320 | 440 | 430 |

